COLLEGE MATHEMATICS

for Business, Economics, Life Sciences, and Social Sciences

BARNETT | ZIEGLER | BYLEEN | STOCKER

fourteenth edition

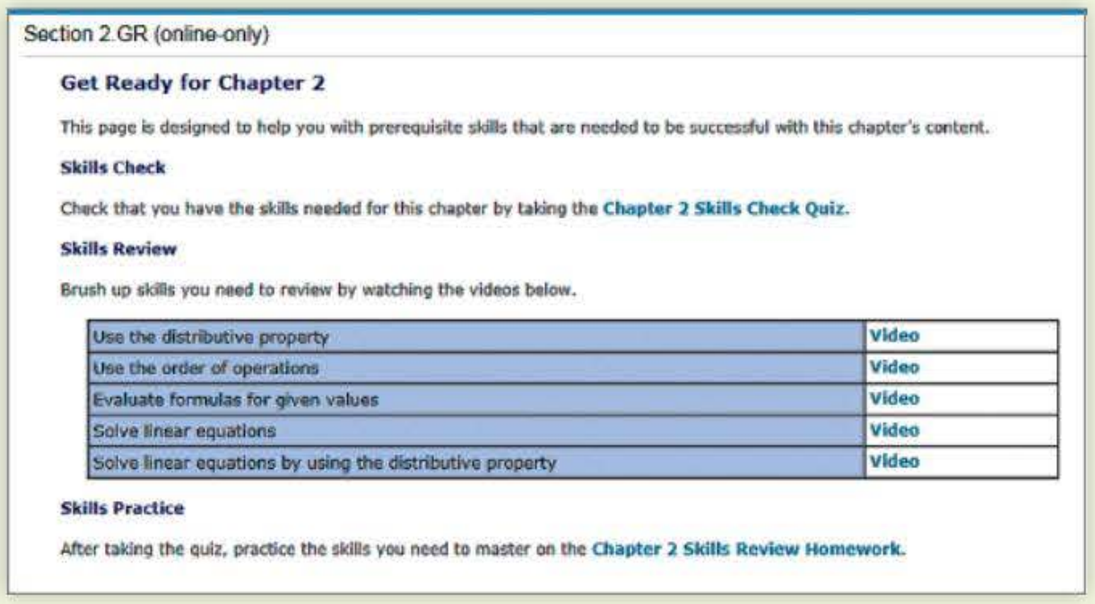
MyLab Math for *College Mathematics for Business, Economics, Life Sciences, and Social Sciences*, 14e

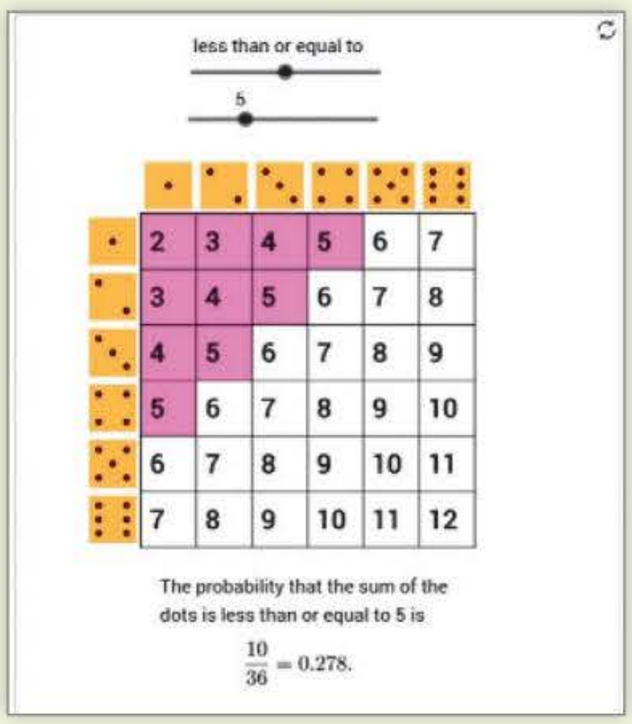
(access code required)

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math delivers assessment, tutorials, and multimedia resources that provide engaging and personalized experiences for each student, so learning can happen in any environment.

Integrated Review

An Integrated Review version of the MyLab Math course contains pre-made, assignable quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified. Each student, therefore, receives just the help that he or she needs—no more, no less.



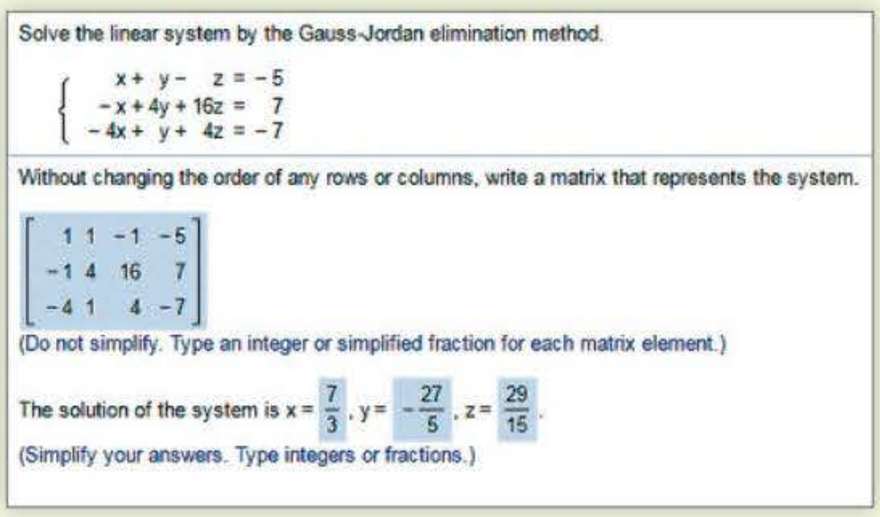


Interactive Figures

A full suite of Interactive Figures has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.

Questions that Deepen Understanding

MyLab Math includes a variety of question types designed to help students succeed in the course. In Setup & Solve questions, students show how they set up a problem as well as the solution, better mirroring what is required on tests. Additional Conceptual Questions provide support for assessing concepts and vocabulary. Many of these questions are application oriented.



pearson.com/mylab/math

fourteenth edition

COLLEGE MATHEMATICS

for Business, Economics, Life Sciences, and Social Sciences

RAYMOND A. BARNETT Merritt College

MICHAEL R. ZIEGLER Marquette University

KARL E. BYLEEN Marquette University

CHRISTOPHER J. STOCKER Marquette University

Pearson

Director, Portfolio Management: Deirdre Lynch
Executive Editor: Jeff Weidenaar
Editorial Assistant: Jennifer Snyder
Content Producers: Sherry Berg, Ron Hampton
Managing Producer: Karen Wernholm
Senior Producer: Stephanie Green
Manager, Courseware QA: Mary Durnwald
Manager, Content Development: Kristina Evans
Product Marketing Manager: Emily Ockay
Field Marketing Manager: Evan St. Cyr
Marketing Assistants: Erin Rush, Shannon McCormack
Senior Author Support/Technology Specialist: Joe Vetere
Manager, Rights and Permissions: Gina Cheselka
Manufacturing Buyer: Carol Melville, LSC Communications
Art Director: Barbara Atkinson
Production Coordination, Composition, and Illustrations: Integra

Cover Image: Tai11/Shutterstock; Digital Storm/Shutterstock

Photo Credits: Page 1: Durk Talsma/Shuttertock; Page 41: Vlad61/Shutterstock; Page 126: Alan Sheldon/Shuttertock; Page 174: Arena Creative/Shutterstock; Page 257: Sspopov/123RF; Page 287: Kustov/Shutterstock; Page 351: PR Image Factory/Shutterstock; Page 394: S Duffett/Shutterstock; Page; 459: Philip Bird LRPS CPAGB/Shutterstock; 548: Razvan Chisu/123RF; 608: Bozulek/Shutterstock; 690: Thomas Koch/Shutterstock; 755: Maxim Petrichuk/Shutterstock; 802: Sport point/Shutterstock

Text Credits: Page 24: biggreenegg.com; Page 28: www.tradeshop.com; Page 29: www.tradeshop.com; Page 30: National Oceanic and Atmospheric Administration; Page 31: Jack Haggerty Forest, Lakehead University, Canada; Page 33: Energy Information Administration; Page 34: Bureau of TransportaLion Statistics; Page 34: Bureau of Transportation Statistics; Page 34: Centers for Disease Control; Page 34: Walmart Stores, Inc.; Page 35: Kagen Research; Page 35: Lakehead University; Page 35: T. Labuza, University of Minnesota; Page 36: Infoplease.com; Page 36: National Center for Education Statistics; Page 36: National Oceanic and Atmospheric Administration; Page 37: www.usda.gov/nass/pubslhistdata.htm; Page 40: Ashland Inc.; Page 40: U.S. Bureau of Labor Statistics; 485: Institute of Chemistry, Macedonia; 522: NCES; 603: FBI Uniform Crime Reporting; 761: U.S. Census Bureau; 763: The World Factbook, CIA; 845: U.S.Census Bureau; 847: FBI; 847: USDA; 848: NASA/GISS; 848: Organic Trade Association; 848: www.Olympic.org

Library of Congress Cataloging-in-Publication Data

Names: Barnett, Raymond A., author. | Ziegler, Michael R., author. | Byleen, Karl E., author. | Stocker, Christopher J., author.
Title: College mathematics for business, economics, life sciences, and social sciences.
Description: Fourteenth edition / Raymond A. Barnett, Michael R. Ziegler, Karl E. Byleen, Christopher J. Stocker. | Boston : Pearson, [2019] | Includes indexes.
Identifiers: LCCN 2017037295| ISBN 9780134674148 | ISBN 0134674146
Subjects: LCSH: Mathematics–Textbooks. | Social sciences–Mathematics–Textbooks. | Biomathematics–Textbooks.
Classification: LCC QA37.3 .B37 2019 | DDC 510–dc23
LC record available at https://lccn.loc.gov/2017037295

1 18

Pearson

Student Edition
ISBN 13: 978-0-13-467414-8
ISBN 10: 0-13-467414-6

Instructor's Edition
ISBN 13: 978-0-13-467610-4
ISBN 10: 0-13-467610-6

CONTENTS

PREFACE

The fourteenth edition of *College Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a two-term (or condensed one-term) course in finite mathematics and calculus for students who have had one or two years of high school algebra or the equivalent. The book's overall approach, refined by the authors' experience with large sections of college freshmen, addresses the challenges of teaching and learning when prerequisite knowledge varies greatly from student to student.

Note that the content of this text is also available in two separate volumes: (a) *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* and (b) *Calculus for Business, Economics, Life Sciences, and Social Sciences, Brief Version*.

The authors had three main goals in writing this text:

1. To write a text that students can easily comprehend
2. To make connections between what students are learning and how they may apply that knowledge
3. To give flexibility to instructors to tailor a course to the needs of their students.

Many elements play a role in determining a book's effectiveness for students. Not only is it critical that the text be accurate and readable, but also, in order for a book to be effective, aspects such as the page design, the interactive nature of the presentation, and the ability to support and challenge all students have an incredible impact on how easily students comprehend the material. Here are some of the ways this text addresses the needs of students at all levels:

- Page layout is clean and free of potentially distracting elements.
- Matched Problems that accompany each of the completely worked examples help students gain solid knowledge of the basic topics and assess their own level of understanding before moving on.
- Review material (Appendix A and Chapters 1 and 2) can be used judiciously to help remedy gaps in prerequisite knowledge.
- A Diagnostic Prerequisite Test prior to Chapter 1 helps students assess their skills, while the Basic Algebra Review in Appendix A provides students with the content they need to remediate those skills.
- Explore and Discuss problems lead the discussion into new concepts or build upon a current topic. They help students of all levels gain better insight into the mathematical concepts through thought-provoking questions that are effective in both small and large classroom settings.
- Instructors are able to easily craft homework assignments that best meet the needs of their students by taking advantage of the variety of types and difficulty levels of the exercises. Exercise sets at the end of each section consist of a Skills Warm-up (four to eight problems that review prerequisite knowledge specific to that section) followed by problems divided into categories A, B, and C by level of difficulty, with level-C exercises being the most challenging.
- The MyLab Math course for this text is designed to help students help themselves and provide instructors with actionable information about their progress. The immediate feedback students receive when doing homework and practice in MyLab Math is invaluable, and the easily accessible eBook enhances student learning in a way that the printed page sometimes cannot.
- Most important, all students get substantial experience in modeling and solving real-world problems through application examples and exercises chosen from business and economics, life sciences, and social sciences. Great care has been

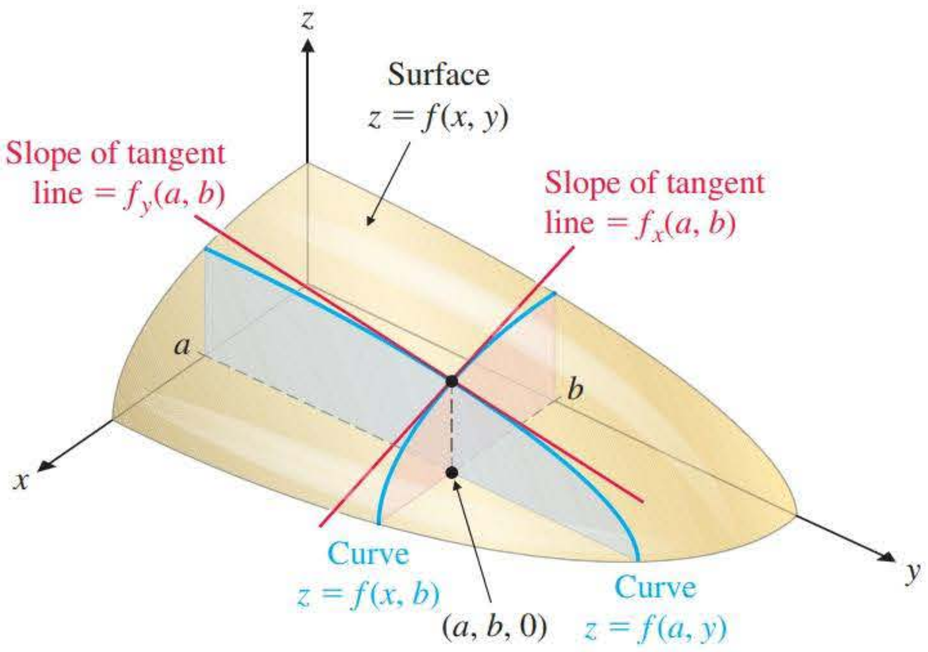
taken to write a book that is mathematically correct, with its emphasis on computational skills, ideas, and problem solving rather than mathematical theory.

- Finally, the choice and independence of topics make the text readily adaptable to a variety of courses.

New to This Edition

Fundamental to a book's effectiveness is classroom use and feedback. Now in its fourteenth edition, this text has had the benefit of a substantial amount of both. Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors. Additionally, we made the following improvements in this edition:

- Redesigned the text in full color to help students better use it and to help motivate students as they put in the hard work to learn the mathematics (because let's face it—a more modern looking book has more appeal).
- Updated graphing calculator screens to TI-84 Plus CE (color edition) format.
- Added *Reminder* features in the side margin to either remind students of a concept that is needed at that point in the book or direct the student back to the section in which it was covered earlier.
- Updated data in examples and exercises. Many modern and student-centered applications have been added to help students see the relevance of the content.
    - Revised all 3-dimensional figures in the text using the latest software. The difference in most cases is stunning, as can be seen in the sample figure here. We took full advantage of these updates to make the figures more effective pedagogically.
    - Analyzed aggregated student performance data and assignment frequency data from MyLab Math for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students.
    - Rewrote and simplified the treatment of cost, revenue, and profit in Section 2.1.
    - In Section 11.5, rewrote Theorem 3 on using the second-derivative test to find absolute extrema, making it applicable to more general intervals.
    - In Section 13.2, rewrote the material on the future value of a continuous income stream to provide a more intuitive and less technical treatment.
    - Added more than 1,000 new exercises throughout the text.



New to MyLab Math

Many improvements have been made to the overall functionality of MyLab Math since the previous edition. However, beyond that, we have also increased and improved the content specific to this text.

- Instructors now have **more exercises** than ever to choose from in assigning homework. Most new questions are application-oriented. There are approximately 6,240 assignable exercises in MyLab Math for this text. New exercise types include:
    - **Additional Conceptual Questions** provide support for assessing concepts and vocabulary. Many of these questions are application-oriented.
    - **Setup & Solve** exercises require students to show how they set up a problem as well as the solution, better mirroring what is required of students on tests.
- The **Guide to Video-Based Assignments** shows which MyLab Math exercises can be assigned for each video. (All videos are also assignable.) This resource is handy for online or flipped classes.

- The ***Note-Taking Guide*** provides support for students as they take notes in class. The Guide includes definitions, theorems, and statements of examples but has blank space for students to write solutions to examples and sample problems. The Note-Taking Guide corresponds to the Lecture PowerPoints that accompany the text. The Guide can be downloaded in PDF or Word format from within MyLab Math.
- A full suite of **Interactive Figures** has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.
- An **Integrated Review** version of the MyLab Math course contains premade quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified.
- **Study Skills Modules** help students with the life skills that can make the difference between passing and failing.
- **MathTalk and StatTalk videos** highlight applications of the content of the course to business. The videos are supported by assignable exercises.
- The ***Graphing Calculator Manual*** and ***Excel Spreadsheet Manual***, both specific to this course, have been updated to support the TI-84 Plus CE (color edition) and Excel 2016, respectively. Both manuals also contain additional topics to support the course. These manuals are within the Tools for Success tab.
- We heard from users that the Annotated Instructor's Edition for the previous edition required too much flipping of pages to find answers, so MyLab Math now contains a downloadable **Instructor's Answers document**—*with all answers in one place*. (This augments the downloadable *Instructor's Solutions Manual*, which contains even-numbered solutions.)

Trusted Features

- **Emphasis and Style**—As was stated earlier, this text is written for student comprehension. To that end, the focus has been on making the book both mathematically correct and accessible to students. Most derivations and proofs are omitted, except where their inclusion adds significant insight into a particular concept as the emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. General concepts and results are typically presented only after particular cases have been discussed.
- **Design**—One of the hallmark features of this text is the clean, straightforward design of its pages. Navigation is made simple with an obvious hierarchy of key topics and a judicious use of call-outs and pedagogical features. A functional use of color improves the clarity of many illustrations, graphs, and explanations, and guides students through critical steps (see pages 59 and 60).
- **Examples**—More than 490 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. The examples are annotated using blue text to the right of each step, and the problem-solving steps are clearly identified. To give students extra help in working through examples, dashed boxes are used to enclose steps that are usually performed mentally and rarely mentioned in other books (see Example 7 on page 7). Though some students may not need these additional steps, many will appreciate the fact that the authors do not assume too much in the way of prior knowledge.
- **Matched Problems**—Each example is followed by a similar Matched Problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

- **Explore and Discuss**—Most every section contains Explore and Discuss problems at appropriate places to encourage students to think about a relationship or process before a result is stated or to investigate additional consequences of a development in the text (see pages 13 and 17). This serves to foster critical thinking and communication skills. The Explore and Discuss material can be used for in-class discussions or out-of-class group activities and is effective in both small and large class settings.
- **Exercise Sets**—The book contains over 6,700 carefully selected and graded exercises. Many problems have multiple parts, significantly increasing the total number of exercises. Writing exercises, indicated by the icon ✎, provide students with an opportunity to express their understanding of the topic in writing. Answers to all odd-numbered problems are in the back of the book. Exercises are paired so that consecutive odd- and even-numbered exercises are of the same type and difficulty level. Exercise sets are structured to facilitate crafting just the right assignment for students:
  - **Skills Warm-up** exercises, indicated by W, review key prerequisite knowledge.
  - **Graded exercises**: Levels A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) make it easy for instructors to create assignments that are appropriate for their classes.
  - **Applications** conclude almost every exercise set. These exercises are labeled with the type of application to make it easy for instructors to select the right exercises for their audience.
- **Applications**—A major objective of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Index of Applications at the back of the book). Almost every exercise set contains application problems, including applications from business and economics, life sciences, and social sciences. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems inspired by professional journals and books. No specialized experience is required to solve any of the application problems.
- **Graphing Calculator and Spreadsheets**—Although access to a graphing calculator or spreadsheets is not assumed, it is likely that many students will want to make use of this technology. To assist these students, optional graphing calculator and spreadsheet activities are included in appropriate places. These include brief discussions in the text, examples or portions of examples solved on a graphing calculator or spreadsheet, and exercises for the students to solve. For example, linear regression is introduced in Section 1.3, and regression techniques on a graphing calculator are used at appropriate points to illustrate mathematical modeling with real data. All the optional graphing calculator material is clearly identified with the icon and can be omitted without loss of continuity, if desired. Graphing calculator screens displayed in the text are actual output from the TI-84 Plus CE (color edition) graphing calculator.

Additional Pedagogical Features

The following features, while helpful to any student, are particularly helpful to students enrolled in a large classroom setting where access to the instructor is more challenging or just less frequent. These features provide much-needed guidance for students as they tackle difficult concepts.

- **Call-out boxes** highlight important definitions, results, and step-by-step processes (see pages 18, 62, and 69).
- **Caution** statements appear throughout the text where student errors often occur (see pages 50 and 115).
- **Conceptual Insights**, appearing in nearly every section, often make explicit connections to previous knowledge but sometimes encourage students to think beyond the particular skill they are working on and attain a more enlightened view of the concepts at hand (see pages 58 and 69).
- **Diagnostic Prerequisite Test**, located on pages xix and xx, provides students with a tool to assess their prerequisite skills prior to taking the course. The **Basic Algebra Review**, in Appendix A, provides students with seven sections of content to help them remediate in specific areas of need. Answers to the Diagnostic Prerequisite Test are at the back of the book and reference specific sections in the Basic Algebra Review or Chapters 1 and 2 for students to use for remediation.
- **Chapter Reviews**—Often it is during the preparation for a chapter exam that concepts gel for students, making the chapter review material particularly important. The chapter review sections in this text include a comprehensive summary of important terms, symbols, and concepts, keyed to completely worked examples, followed by a comprehensive set of Review Exercises. Answers to Review Exercises are included at the back of the book; each answer contains a reference to the section in which that type of problem is discussed so students can remediate any deficiencies in their skills on their own.

Content

The text begins with the development of a library of elementary functions in Chapters 1 and 2, including their properties and applications. Many students will be familiar with most, if not all, of the material in these introductory chapters. Depending on students' preparation and the course syllabus, an instructor has several options for using the first two chapters, including the following:

(i) Skip Chapters 1 and 2 and refer to them only as necessary later in the course;
(ii) Cover Chapter 1 quickly in the first week of the course, emphasizing price–demand equations, price–supply equations, and linear regression, but skip Chapter 2;
(iii) Cover Chapters 1 and 2 systematically before moving on to other chapters.

The finite mathematics material can be thought of as four units:

1. Mathematics of finance (Chapter 3)
2. Linear algebra, including matrices, linear systems, and linear programming (Chapters 4, 5, and 6)
3. Probability (Chapters 7 and 8)
4. Applications of linear algebra and probability to Markov chains (Chapter 15)

The first three units are independent of each other, while the fourth unit is dependent on Chapters 4 and 8.

Chapter 3 presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix B.1 addresses arithmetic and geometric sequences and can be covered in conjunction with this chapter, if desired.

Chapter 4 covers linear systems and matrices with an emphasis on using row operations and Gauss–Jordan elimination to solve systems and to find matrix inverses. This chapter also contains numerous applications of mathematical modeling using systems and matrices. To assist students in formulating solutions, all answers at the

back of the book for application exercises in Sections 4.3, 4.5, and the chapter Review Exercises contain both the mathematical model and its solution. The row operations discussed in Sections 4.2 and 4.3 are required for the simplex method in Chapter 6. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 15.

Chapters 5 and 6 provide a broad and flexible coverage of linear programming. Chapter 5 covers two-variable graphing techniques. Instructors who wish to emphasize linear programming techniques can cover the basic simplex method in Sections 6.1 and 6.2 and then discuss either or both of the following: the dual method (Section 6.3) and the big M method (Section 6.4). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 6.2–6.4, and either omit the solution or use software to find the solution. To facilitate this approach, all answers at the back of the book for application exercises in Sections 6.2–6.4 and the chapter Review Exercises contain both the mathematical model and its solution.

Chapter 7 provides a foundation for probability with a treatment of logic, sets, and counting techniques.

Chapter 8 covers basic probability, including Bayes' formula and random variables.

The calculus material consists of differential calculus (Chapters 9–11), integral calculus (Chapters 12 and 13), and multivariable calculus (Chapter 14). In general, Chapters 9–12 must be covered in sequence; however, certain sections can be omitted or given brief treatments, as pointed out in the discussion that follows.

Chapter 9 introduces the derivative. The first three sections cover limits (including infinite limits and limits at infinity), continuity, and the limit properties that are essential to understanding the definition of the derivative in 9.4. The remaining sections cover basic rules of differentiation, differentials, and applications of derivatives in business and economics. The interplay between graphical, numerical, and algebraic concepts is emphasized here and throughout the text.

In **Chapter 10** the derivatives of exponential and logarithmic functions are obtained before the product rule, quotient rule, and chain rule are introduced. Implicit differentiation is introduced in Section 10.5 and applied to related rates problems in Section 10.6. Elasticity of demand is introduced in Section 10.7. The topics in these last three sections of Chapter 10 are not referred to elsewhere in the text and can be omitted.

Chapter 11 focuses on graphing and optimization. The first two sections cover first-derivative and second-derivative graph properties. L'Hôpital's rule is discussed in Section 11.3. A graphing strategy is introduced in Section 11.2 and developed in Section 11.4. Optimization is covered in Sections 11.5 and 11.6, including examples and problems involving endpoint solutions.

Chapter 12 introduces integration. The first two sections cover antidifferentiation techniques essential to the remainder of the text. Section 12.3 discusses some applications involving differential equations that can be omitted. The definite integral is defined in terms of Riemann sums in Section 12.4 and the Fundamental Theorem of Calculus is discussed in Section 12.5. As before, the interplay between graphical, numerical, and algebraic properties is emphasized.

Chapter 13 covers additional integration topics and is organized to provide maximum flexibility for the instructor. The first section extends the area concepts introduced in Chapter 13 to the area between two curves and related applications. Section 13.2 covers three more applications of integration, and Sections 13.3 and 13.4 deal with additional methods of integration, including integration by parts, the trapezoidal rule, and Simpson's rule. Any or all of the topics in Chapter 13 can be omitted.

Chapter 14 deals with multivariable calculus. The first five sections can be covered any time after Section 11.6 has been completed. Sections 14.6 and 14.7 require the integration concepts discussed in Chapter 12.

Chapter 15 on Markov chains is available online at `goo.gl/8SZkyn` and provides a worthwhile and interesting application of finite mathematics. Full support for this chapter is provided in MyLab Math.

Appendix A contains a concise review of basic algebra that may be covered as part of the course or referenced as needed. As mentioned previously, **Appendix B** (online at `goo.gl/mjbXrG`) contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

Accuracy Check—Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error free. If an error should be found, the authors would be grateful if notification were sent to Karl E. Byleen, 9322 W. Garden Court, Hales Corners, WI 53130; or by e-mail to kbyleen@wi.rr.com.

Acknowledgments

In addition to the authors, many others are involved in the successful publication of a book. We wish to thank the following reviewers:

Ebrahim Ahmadizadeh, *Northampton Community College*
Simon Aman, *Truman College*
Ben Anilus, *Roger Williams University*
Paula F. Bailey, *Roger Williams University*
Mary Ann Barber, *University of North Texas*
B. Bruce Bare, *University of Washington*
Tammy Barker, *Hillsborough Community College*
Sandra Belcher, *Midwestern State University*
Chris Bendixen, *Lake Michigan College*
Clark Bennett, *University of South Dakota*
Brian C. Bright, *Ivy Tech Community College*
Ronnie L. Brown, *American University*
William Chin, *DePaul University*
Dee Cochran, *Indiana University, Southeast*
Christine Curtis, *Hillsborough Community College*
Lauren Fern, *University of Montana*
Toni Fountain, *Chattanooga State Community College*
Sonya Geisel, *Arizona State University*
Caleb Grisham, *National Park College*
Jeffrey Hakim, *American University*
Robert G. Hinkle, *Germanna Community College*
Mark Hunacek, *Iowa State University*
Patricia Ann Hussey, *Triton College*
Chuck Jackson, *Lone Star College, Kingwood*
Doug Jones, *Tallahassee Community College*
Matthew E. Lathrop, *Heartland Community College*
Pat LaVallo, *Mission College*
Sheila C. Luke, *Colorado Technical University*
Mari M. Menard, *Lone Star College, Kingwood*
Faramarz Mokhtari, *Montgomery College, Rockville*
Luis Molina, *Lone Star College, CyFair*
Jimmy Mopecha, *Delaware Valley University*
Quinn A. Morris, *University of North Carolina, Greensboro*
Kayo Motomiya, *Bunker Hill Community College*
Lyn A. Noble, *Florida State College at Jacksonville*

Tuyet Pham, *Kent State University*
Stephen Proietti, *Northern Essex Community College*
Michael Puente, *Richland College*
Rick Pugsley, *Ivy Tech Community College*
Douglas Ray, *Texas State University*
Jean Schneider, *Boise State University*
Jacob Skala, *Wichita State University*
Frances Smith, *Richland College*
Sid S. Syam, *Marquette University*
John C. Treacy, *Ivy Tech Community College*
Brent Wessel, *St. Louis University*
Bashkim Zendeli, *Lawrence Technological University*

The following faculty members provided direction on the development of the MyLab Math course for this edition:

Wael Abushammala, *University of Houston*
Emil D. Akli, *Harold Washington College*
L. Wyatt Alverson, II, *Murray State University*
Mary Ann Barber, *University of North Texas*
Clark Bennett, *University of South Dakota*
Latrice N. Bowman, *University of Alaska, Fairbanks*
Ronnie L. Brown, *University of Baltimore*
Debra Bryant, *Tennessee Tech University*
David Busekist, *Southeastern Louisiana University*
Kristen A. Ceballos, *Southern Illinois University, Carbondale*
Burak Reis Cebecioglu, *Grossmont College*
Brandi L. Cline, *Lone Star College, Tomball*
James Colton, *Shaker High School*
Nene Coulibaly, *Texas A&M University, Corpus Christi*
Christine Curtis, *Hillsborough Community College, Dale Mabry*
Kristel Ehrhardt, *Montgomery College, Germantown*
Natalia Gavryshova, *College of San Mateo*
Nicole Gesiskie, *Luzerne County Community College*
Liz Hawk, *Eastbrook High School*
Natalie Herndon, *Southwest Tennessee State University*
Robert G. Hinkle, *Germanna Community College*
Jo Beth Horney, *South Plains College*
Abushieba Ibrahim, *Broward College*
Elaine Jadacki, *Montgomery College*
Ilija Jegdić, *Texas Southern University*
Katarina Jegdić, *University of Houston*
Kiandra Johnson, *Spelman College*
Alan Kunkle, *Ivy Tech Community College, Kokomo*
Steve Lam, *Guam Community College*
Jiarong Li, *Harold Washington College*
Sarah Lynch, *University of North Texas*
Mari M. Menard, *Lone Star College, Kingwood*
Gabriel Mendoza, *El Paso Community College*
Faramarz Mokhtari, *Montgomery College, Rockville*
Jimmy Mopecha, *Delaware Valley University*
Jillian Morrison, *Washington State University*
Mike Panahi, *El Centro College*
Armando I. Perez, *Laredo Community College*
Cristian Sabau, *Broward College*
Leonel Saldivar, *Mission Early College High School*
Terri Seiver, *San Jacinto College*
Ed W. Stringer, III, *Florida State College at Jacksonville*

James Tian, *Hampton University*
Jack Wagner, *Fordham University*
Dennis Wolf, *Indiana University, South Bend*
Dinesh S. Yadav, *Richland College*
Pengfei Yao, *Southwest Tennessee Community College*

We also express our thanks to John Samons, Patricia Nelson, and Damon Demas for providing a careful and thorough accuracy check of the text, problems, and answers. Our thanks to Garret Etgen, John Samons, Salvatore Sciandra, Victoria Baker, Ben Rushing, and Stela Pudar-Hozo for developing the supplemental materials so important to the success of a text. And finally, thanks to all the people at Pearson and Integra who contributed their efforts to the production of this book.

MyLab Math Online Course for *College Mathematics for Business, Economics, Life Sciences, and Social Sciences,* 14e (access code required)

MyLab™ Math is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method, each text's flavor and approach are tightly integrated throughout the accompanying MyLab Math course, making learning the material as seamless as possible.

Preparedness

One of the biggest challenges in applied math courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with just-in-time remediation and key-concept review.

NEW! Integrated Review Course

An Integrated Review version of the MyLab Math course contains premade, assignable quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified. Each student, therefore, receives just the help that he or she needs—no more, no less.

NEW! Study Skills Modules

Study skills modules help students with the life skills that can make the difference between passing and failing.

Developing Deeper Understanding

MyLab Math provides content and tools that help students build a deeper understanding of course content than would otherwise be possible.

Exercises with Immediate Feedback

Homework and practice exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes the optional learning aids Help Me Solve This, View an Example, videos, and/or the eText.

pearson.com/mylab/math

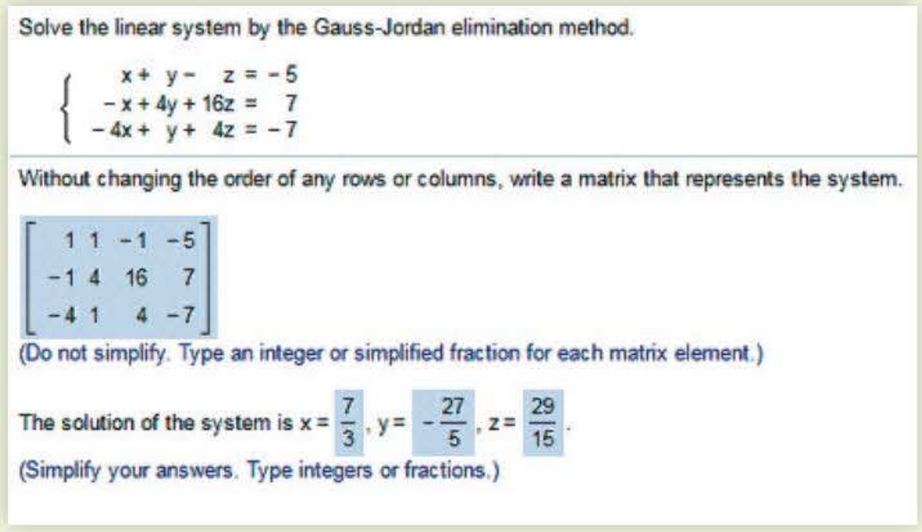
Pearson MyLab

NEW! Additional Conceptual Questions

Additional Conceptual Questions provide support for assessing concepts and vocabulary. Many of these questions are application-oriented. They are clearly labeled "Conceptual" in the Assignment Manager.

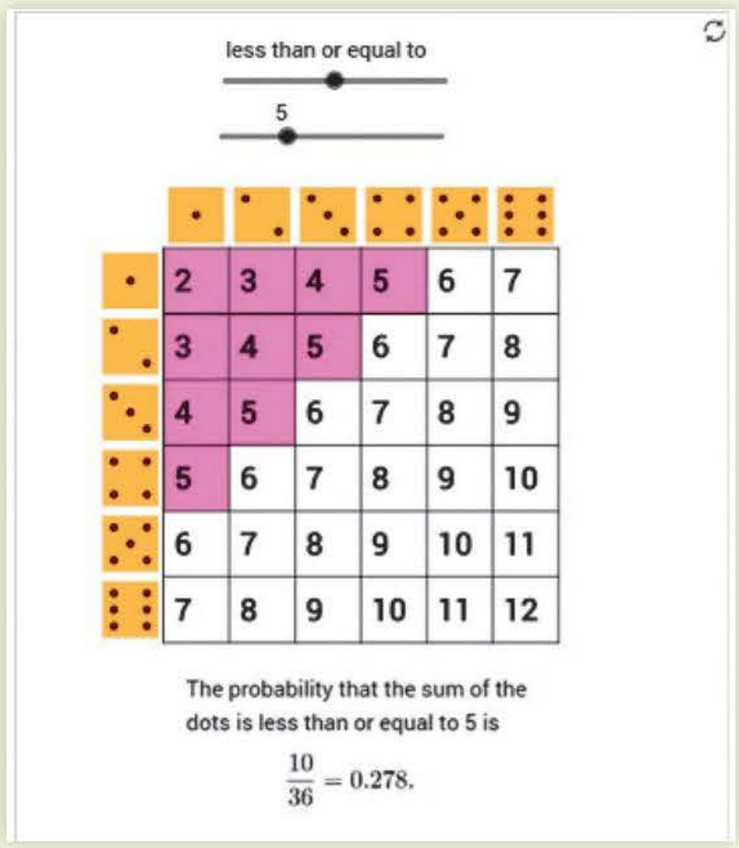
NEW! Setup & Solve Exercises

These exercises require students to show how they set up a problem as well as the solution, better mirroring what is required on tests.



NEW! Interactive Figures

A full suite of Interactive Figures has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They are designed to be used in lecture as well as by students independently.



Instructional Videos

Every example in the text has an instructional video tied to it that can be used as a learning aid or for self-study. MathTalk videos were added to highlight business applications to the course content, and a Guide to Video-Based Assignments shows which MyLab Math exercises can be assigned for each video.

NEW! Note-Taking Guide (downloadable)

These printable sheets, developed by Ben Rushing (Northwestern State University) provide support for students as they take notes in class. They include preprinted definitions, theorems, and statements of examples but have blank space for students to write solutions to examples and sample problems. The *Note-Taking Guide* corresponds to the Lecture PowerPoints that accompany the text. The *Guide* can be downloaded in PDF or Word format from within MyLab Math from the Tools for Success tab.

pearson.com/mylab/math

Pearson MyLab

Graphing Calculator and Excel Spreadsheet Manuals (downloadable)

Graphing Calculator Manual by Chris True, University of Nebraska
Excel Spreadsheet Manual by Stela Pudar-Hozo, Indiana University–Northwest
These manuals, both specific to this course, have been updated to support the TI-84 Plus CE (color edition) and Excel 2016, respectively. Instructions are ordered by mathematical topic. The files can be downloaded from within MyLab Math from the Tools for Success tab.

Student's Solutions Manual (softcover and downloadable)

ISBN: 0-13-467741-2 • 978-0-13-467741-5
Written by John Samons (Florida State College), the *Student's Solutions Manual* contains worked-out solutions to all the odd-numbered exercises. This manual is available in print and can be downloaded from within MyLab Math within the Chapter Contents tab.

A Complete eText

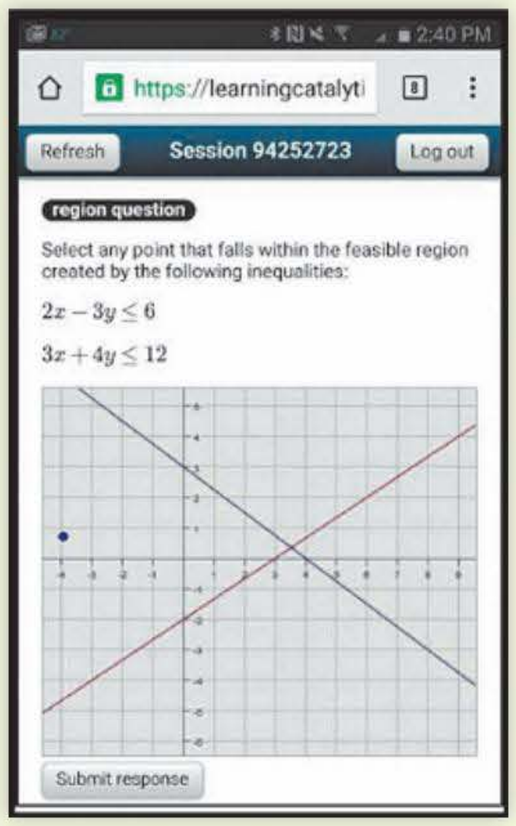
Students get unlimited access to the eText within any MyLab Math course using that edition of the textbook. The Pearson eText app allows existing subscribers to access their titles on an iPad or Android tablet for either online or offline viewing.

Supporting Instruction

MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve students' results at every turn and unparalleled flexibility to create a course tailored to you and your students.

Learning Catalytics™

Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics™ fosters student engagement and peer-to-peer learning with real-time analytics. Access pre-built exercises created specifically for this course.



PowerPoint® Lecture Slides (downloadable)

Classroom presentation slides feature key concepts, examples, and definitions from this text. They are designed to be used in conjunction with the Note-Taking Guide that accompanies the text. They can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearson.com**.

pearson.com/mylab/math

Pearson MyLab

Learning Worksheets

Written by Salvatore Sciandra (Niagara County Community College), these worksheets include key chapter definitions and formulas, followed by exercises for students to practice in class, for homework, or for independent study. They are downloadable as PDFs or Word documents from within MyLab Math.

Comprehensive Gradebook

The gradebook includes enhanced reporting functionality such as item analysis and a reporting dashboard to allow you to efficiently manage your course. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.

TestGen®

TestGen® (**www.pearson.com/testgen**) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson's online catalog, **www.pearson.com**. The questions are also assignable in MyLab Math.

Instructor's Solutions Manual (downloadable)

Written by Garret J. Etgen (University of Houston) and John Samons (Florida State College), the *Instructor's Solutions Manual* contains worked-out solutions to all the even-numbered exercises. It can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearson.com**.

Accessibility

Pearson works continuously to ensure our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, **www.pearson.com/mylab/math/accessibility.**

pearson.com/mylab/math

Diagnostic Prerequisite Test

Work all of the problems in this self-test without using a calculator. Then check your work by consulting the answers in the back of the book. Where weaknesses show up, use the reference that follows each answer to find the section in the text that provides the necessary review.

1. Replace each question mark with an appropriate expression that will illustrate the use of the indicated real number property:

(A) Commutative $(\cdot)$: $x(y + z) = ?$

(B) Associative $(+)$: $2 + (x + y) = ?$

(C) Distributive: $(2 + 3)x = ?$

Problems 2–6 refer to the following polynomials:

(A) $3x - 4$ (B) $x + 2$

(C) $2 - 3x^2$ (D) $x^3 + 8$

2. Add all four.

3. Subtract the sum of (A) and (C) from the sum of (B) and (D).

4. Multiply (C) and (D).

5. What is the degree of each polynomial?

6. What is the leading coefficient of each polynomial?

In Problems 7 and 8, perform the indicated operations and simplify.

7. $5x^2 - 3x[4 - 3(x - 2)]$

8. $(2x + y)(3x - 4y)$

In Problems 9 and 10, factor completely.

9. $x^2 + 7x + 10$

10. $x^3 - 2x^2 - 15x$

11. Write 0.35 as a fraction reduced to lowest terms.

12. Write $\frac{7}{8}$ in decimal form.

13. Write in scientific notation:

(A) 4,065,000,000,000 (B) 0.0073

14. Write in standard decimal form:

(A) 2.55×10^8 (B) 4.06×10^{-4}

15. Indicate true (T) or false (F):

(A) A natural number is a rational number.

(B) A number with a repeating decimal expansion is an irrational number.

16. Give an example of an integer that is not a natural number.

In Problems 17–24, simplify and write answers using positive exponents only. All variables represent positive real numbers.

17. $6(xy^3)^5$

18. $\frac{9u^8v^6}{3u^4v^8}$

19. $(2 \times 10^5)(3 \times 10^{-3})$

20. $(x^{-3}y^2)^{-2}$

21. $u^{5/3}u^{2/3}$

22. $(9a^4b^{-2})^{1/2}$

23. $\frac{5^0}{3^2} + \frac{3^{-2}}{2^{-2}}$

24. $(x^{1/2} + y^{1/2})^2$

In Problems 25–30, perform the indicated operation and write the answer as a simple fraction reduced to lowest terms. All variables represent positive real numbers.

25. $\frac{a}{b} + \frac{b}{a}$

26. $\frac{a}{bc} - \frac{c}{ab}$

27. $\frac{x^2}{y} \cdot \frac{y^6}{x^3}$

28. $\frac{x}{y^3} \div \frac{x^2}{y}$

29. $\dfrac{\frac{1}{7+h} - \frac{1}{7}}{h}$

30. $\dfrac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$

31. Each statement illustrates the use of one of the following real number properties or definitions. Indicate which one.

Commutative $(+, \cdot)$	Associative $(+, \cdot)$	Distributive
Identity $(+, \cdot)$	Inverse $(+, \cdot)$	Subtraction
Division	Negatives	Zero

(A) $(-7) - (-5) = (-7) + [-(-5)]$

(B) $5u + (3v + 2) = (3v + 2) + 5u$

(C) $(5m - 2)(2m + 3) = (5m - 2)2m + (5m - 2)3$

(D) $9 \cdot (4y) = (9 \cdot 4)y$

(E) $\dfrac{u}{-(v - w)} = \dfrac{u}{w - v}$

(F) $(x - y) + 0 = (x - y)$

32. Round to the nearest integer:

(A) $\frac{17}{3}$ (B) $-\frac{5}{19}$

33. Multiplying a number x by 4 gives the same result as subtracting 4 from x. Express as an equation, and solve for x.

34. Find the slope of the line that contains the points $(3, -5)$ and $(-4, 10)$.

35. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the x axis.

36. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the y axis.

In Problems 37 and 38, factor completely.

37. $x^2 - 3xy - 10y^2$

38. $6x^2 - 17xy + 5y^2$

In Problems 39–42, write in the form $ax^p + by^q$ where a, b, p, and q are rational numbers.

39. $\dfrac{3}{x} + 4\sqrt{y}$

40. $\dfrac{8}{x^2} - \dfrac{5}{y^4}$

41. $\dfrac{2}{5x^{3/4}} - \dfrac{7}{6y^{2/3}}$

42. $\dfrac{1}{3\sqrt{x}} + \dfrac{9}{\sqrt[3]{y}}$

In Problems 43 and 44, write in the form $a + b\sqrt{c}$ where a, b, and c are rational numbers.

43. $\dfrac{1}{4 - \sqrt{2}}$

44. $\dfrac{5 - \sqrt{3}}{5 + \sqrt{3}}$

In Problems 45–50, solve for x.

45. $x^2 = 5x$

46. $3x^2 - 21 = 0$

47. $x^2 - x - 20 = 0$

48. $-6x^2 + 7x - 1 = 0$

49. $x^2 + 2x - 1 = 0$

50. $x^4 - 6x^2 + 5 = 0$

This page intentionally left blank

1 Linear Equations and Graphs

Introduction

How far will a glacier advance or retreat in the next ten years? The key to answering such a question, and to making other climate-related predictions, is mathematical modeling. In Chapter 1, we study one of the simplest mathematical models, a linear equation. We introduce a technique called linear regression to construct mathematical models from numerical data. We use mathematical models to predict average annual temperature and average annual precipitation (see Problems 23 and 24 in Section 1.3), the atmospheric concentration of carbon dioxide, the consumption of fossil fuels, and many other quantities in business, economics, life sciences, and social sciences.

1.1 Linear Equations and Inequalities

- Linear Equations
- Linear Inequalities
- Applications

The equation

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

and the inequality

$$\frac{x}{2} + 2(3x - 1) \geq 5$$

are both first degree in one variable. In general, a **first-degree**, or **linear**, **equation** in one variable is any equation that can be written in the form

$$\textbf{Standard form: } ax + b = 0 \quad a \neq 0 \tag{1}$$

If the equality symbol, $=$, in (1) is replaced by $<$, $>$, $\leq$, or $\geq$, the resulting expression is called a **first-degree**, or **linear**, **inequality**.

A **solution** of an equation (or inequality) involving a single variable is a number that, when substituted for the variable, makes the equation (or inequality) true. The set of all solutions is called the **solution set**. To **solve an equation** (or inequality) means to find its solution set.

Knowing what is meant by the solution set is one thing; finding it is another. We start by recalling the idea of equivalent equations and equivalent inequalities. If we perform an operation on an equation (or inequality) that produces another equation (or inequality) with the same solution set, then the two equations (or inequalities) are said to be **equivalent**. The basic idea in solving equations or inequalities is to perform operations that produce simpler equivalent equations or inequalities and to continue the process until we obtain an equation or inequality with an obvious solution.

Linear Equations

Linear equations are generally solved using the following equality properties.

THEOREM 1 Equality Properties

An equivalent equation will result if

1. The same quantity is added to or subtracted from each side of a given equation.
2. Each side of a given equation is multiplied by or divided by the same nonzero quantity.

EXAMPLE 1 Solving a Linear Equation Solve and check:

$$8x - 3(x - 4) = 3(x - 4) + 6$$

SOLUTION

$$\begin{aligned}
8x - 3(x - 4) &= 3(x - 4) + 6 && \text{Use the distributive property.}\\
8x - 3x + 12 &= 3x - 12 + 6 && \text{Combine like terms.}\\
5x + 12 &= 3x - 6 && \text{Subtract } 3x \text{ from both sides.}\\
2x + 12 &= -6 && \text{Subtract 12 from both sides.}\\
2x &= -18 && \text{Divide both sides by 2.}\\
x &= -9
\end{aligned}$$

CHECK

$$\begin{aligned} 8x - 3(x - 4) &= 3(x - 4) + 6 \\ 8(-9) - 3[(-9) - 4] &\stackrel{?}{=} 3[(-9) - 4] + 6 \\ -72 - 3(-13) &\stackrel{?}{=} 3(-13) + 6 \\ -33 &\stackrel{\checkmark}{=} -33 \end{aligned}$$

Matched Problem 1 Solve and check: $3x - 2(2x - 5) = 2(x + 3) - 8$

Explore and Discuss 1

According to equality property 2, multiplying both sides of an equation by a nonzero number always produces an equivalent equation. What is the smallest positive number that you could use to multiply both sides of the following equation to produce an equivalent equation without fractions?

$$\frac{x + 1}{3} - \frac{x}{4} = \frac{1}{2}$$

EXAMPLE 2 **Solving a Linear Equation** Solve and check: $\dfrac{x + 2}{2} - \dfrac{x}{3} = 5$

SOLUTION What operations can we perform on

$$\frac{x + 2}{2} - \frac{x}{3} = 5$$

to eliminate the denominators? If we can find a number that is exactly divisible by each denominator, we can use the multiplication property of equality to clear the denominators. The LCD (least common denominator) of the fractions, 6, is exactly what we are looking for! Actually, any common denominator will do, but the LCD results in a simpler equivalent equation. So we multiply both sides of the equation by 6:

$$6\left(\frac{x + 2}{2} - \frac{x}{3}\right) = 6 \cdot 5$$

$$\overset{3}{\cancel{6}} \cdot \frac{(x + 2)}{\underset{1}{\cancel{2}}} - \overset{2}{\cancel{6}} \cdot \frac{x}{\underset{1}{\cancel{3}}} = 30$$

$$\begin{aligned} 3(x + 2) - 2x &= 30 && \text{Use the distributive property.} \\ 3x + 6 - 2x &= 30 && \text{Combine like terms.} \\ x + 6 &= 30 && \text{Subtract 6 from both sides.} \\ x &= 24 \end{aligned}$$

CHECK

$$\begin{aligned} \frac{x + 2}{2} - \frac{x}{3} &= 5 \\ \frac{24 + 2}{2} - \frac{24}{3} &\stackrel{?}{=} 5 \\ 13 - 8 &\stackrel{?}{=} 5 \\ 5 &\stackrel{\checkmark}{=} 5 \end{aligned}$$

Matched Problem 2 Solve and check: $\dfrac{x + 1}{3} - \dfrac{x}{4} = \dfrac{1}{2}$

Reminder

Dashed boxes are used throughout the book to denote steps that are usually performed mentally.

In many applications of algebra, formulas or equations must be changed to alternative equivalent forms. The following example is typical.

EXAMPLE 3 **Solving a Formula for a Particular Variable** If you deposit a principal P in an account that earns simple interest at an annual rate r, then the amount A in the account after t years is given by $A = P + Prt$. Solve for

(A) r in terms of A, P, and t

(B) P in terms of A, r, and t

SOLUTION (A)

$$A = P + Prt \qquad \text{Reverse equation.}$$

$$P + Prt = A \qquad \text{Subtract } P \text{ from both sides.}$$

$$Prt = A - P \qquad \text{Divide both members by } Pt.$$

$$r = \frac{A - P}{Pt}$$

(B)

$$A = P + Prt \qquad \text{Reverse equation.}$$

$$P + Prt = A \qquad \text{Factor out } P \text{ (note the use of the distributive property).}$$

$$P(1 + rt) = A \qquad \text{Divide by } (1 + rt).$$

$$P = \frac{A}{1 + rt}$$

Matched Problem 3 If a cardboard box has length L, width W, and height H, then its surface area is given by the formula $S = 2LW + 2LH + 2WH$. Solve the formula for

(A) L in terms of S, W, and H (B) H in terms of S, L, and W

Linear Inequalities

Before we start solving linear inequalities, let us recall what we mean by $<$ (less than) and $>$ (greater than). If a and b are real numbers, we write

$$a < b \quad a \text{ is less than } b$$

if there exists a positive number p such that $a + p = b$. Certainly, we would expect that if a positive number was added to any real number, the sum would be larger than the original. That is essentially what the definition states. If $a < b$, we may also write

$$b > a \quad b \text{ is greater than } a.$$

EXAMPLE 4 **Inequalities** Replace each question mark with either $<$ or $>$.

(A) $3 \;?\; 5$ (B) $-6 \;?\; -2$ (C) $0 \;?\; -10$

SOLUTION

(A) $3 < 5$ because $3 + 2 = 5$.

(B) $-6 < -2$ because $-6 + 4 = -2$.

(C) $0 > -10$ because $-10 < 0$ (because $-10 + 10 = 0$).

Matched Problem 4 Replace each question mark with either $<$ or $>$.

(A) $2 \;?\; 8$ (B) $-20 \;?\; 0$ (C) $-3 \;?\; -30$

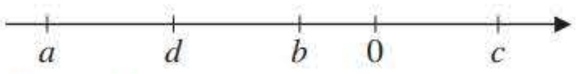


Figure 1 $a < b, c > d$

The inequality symbols have a very clear geometric interpretation on the real number line. If $a < b$, then a is to the left of b on the number line; if $c > d$, then c is to the right of d on the number line (Fig. 1). Check this geometric property with the inequalities in Example 4.

Explore and Discuss 2

Replace ? with $<$ or $>$ in each of the following:

(A) $-1 \;?\; 3$ and $2(-1) \;?\; 2(3)$

(B) $-1 \;?\; 3$ and $-2(-1) \;?\; -2(3)$

(C) $12 \;?\; -8$ and $\dfrac{12}{4} \;?\; \dfrac{-8}{4}$

(D) $12 \;?\; -8$ and $\dfrac{12}{-4} \;?\; \dfrac{-8}{-4}$

Based on these examples, describe the effect of multiplying both sides of an inequality by a number.

The procedures used to solve linear inequalities in one variable are almost the same as those used to solve linear equations in one variable, but with one important exception, as noted in item 3 of Theorem 2.

THEOREM 2 Inequality Properties

An equivalent inequality will result, and the **sense or direction will remain the same**, if each side of the original inequality

1. has the same real number added to or subtracted from it.
2. is multiplied or divided by the same *positive* number.

An equivalent inequality will result, and the **sense or direction will reverse**, if each side of the original inequality

3. is multiplied or divided by the same *negative* number.

Note: Multiplication by 0 and division by 0 are not permitted.

Therefore, we can perform essentially the same operations on inequalities that we perform on equations, with the exception that **the sense of the inequality reverses if we multiply or divide both sides by a negative number**. Otherwise, the sense of the inequality does not change. For example, if we start with the true statement

$$-3 > -7$$

and multiply both sides by 2, we obtain

$$-6 > -14$$

and the sense of the inequality stays the same. But if we multiply both sides of $-3 > -7$ by -2, the left side becomes 6 and the right side becomes 14, so we must write

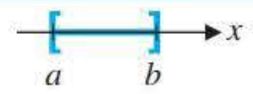
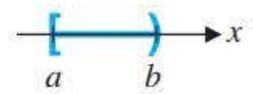
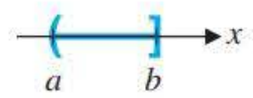
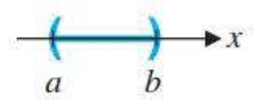
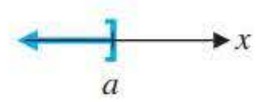
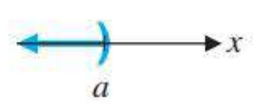
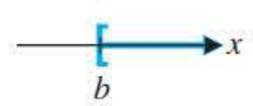
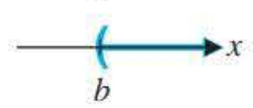
$$6 < 14$$

to have a true statement. The sense of the inequality reverses.

If $a < b$, the **double inequality** $a < x < b$ means that $\boldsymbol{a < x}$ **and** $\boldsymbol{x < b}$; that is, x is between a and b. **Interval notation** is also used to describe sets defined by inequalities, as shown in Table 1.

The numbers a and b in Table 1 are called the **endpoints** of the interval. An interval is **closed** if it contains all its endpoints and **open** if it does not contain any of its endpoints. The intervals $[a, b]$, $(-\infty, a]$, and $[b, \infty)$ are closed, and the intervals (a, b), $(-\infty, a)$, and (b, ∞) are open. Note that the symbol ∞ (read infinity) is not a number. When we write $[b, \infty)$, we are simply referring to the interval that starts at b and continues indefinitely to the right. We never refer to ∞ as an endpoint, and we never write $[b, \infty]$. The interval $(-\infty, \infty)$ is the entire real number line.

Table 1 Interval Notation

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \leq x \leq b$	
$[a, b)$	$a \leq x < b$	
$(a, b]$	$a < x \leq b$	
(a, b)	$a < x < b$	
$(-\infty, a]$	$x \leq a$	
$(-\infty, a)$	$x < a$	
$[b, \infty)$	$x \geq b$	
(b, ∞)	$x > b$	

Note that an endpoint of a line graph in Table 1 has a square bracket through it if the endpoint is included in the interval; a parenthesis through an endpoint indicates that it is not included.

CONCEPTUAL INSIGHT

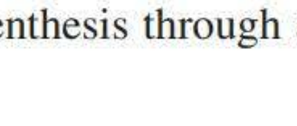
The notation $(2, 7)$ has two common mathematical interpretations: the ordered pair with first coordinate 2 and second coordinate 7, and the open interval consisting of all real numbers between 2 and 7. The choice of interpretation is usually determined by the context in which the notation is used. The notation $(2, -7)$ could be interpreted as an ordered pair but not as an interval. In interval notation, the left endpoint is always written first. So, $(-7, 2)$ is correct interval notation, but $(2, -7)$ is not.

EXAMPLE 5 **Interval and Inequality Notation, and Line Graphs**

(A) Write $[-2, 3)$ as a double inequality and graph.

(B) Write $x \geq -5$ in interval notation and graph.

SOLUTION

(A) $[-2, 3)$ is equivalent to $-2 \leq x < 3$. 

(B) $x \geq -5$ is equivalent to $[-5, \infty)$.

Matched Problem 5

(A) Write $(-7, 4]$ as a double inequality and graph.

(B) Write $x < 3$ in interval notation and graph.

Explore and Discuss 3

The solution to Example 5B shows the graph of the inequality $x \geq -5$. What is the graph of $x < -5$? What is the corresponding interval? Describe the relationship between these sets.